A TECHNIQUE FOR STABILIZING MICROWAVE OSCILLATORS

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Summary

The stabilization of microwave oscillators whose short term deviations in frequency are in the order of one part in 10^8 is discussed in this article. The control system, which consists of a dual mode reference cavity and feedback amplifier, is applied to a reflex klystron oscillator.

Circuit analysis and practical design considerations are presented. A practical method of measuring the actual frequency stability is given.

Introduction

Microwave oscillators may be highly stabilized in frequency, with short time deviations of one part in 10^8, by taking advantage of feedback and two properties specific to the microwave region only: high Q's of the order of several thousands are readily obtainable and small changes on the reflector voltage (order of a volt) of a klystron may change the frequency considerably (order of megacycles). A typical Stable Local Oscillator (Stalo) consists of a stabilizing monitoring loop involving a reflex klystron oscillator, a dual mode reference cavity and a feedback amplifier. The system is tunable.

Drift which would occur because of changes in environmental conditions, such as changes in power line voltage or temperature change of the klystron, is effectively reduced by a factor equal to the loop gain. The dual mode reference cavity itself may be made of invar or may be temperature compensated. Hence, under normal operating conditions, the residual drift may be extremely small (its exact magnitude depending on actual ambient conditions).

The following technique is applicable whenever a reflex klystron is used as the oscillator.

I. General Theory of Operation

The system operates in the following manner (see Fig. 1). A portion of the klystron rf power output is fed into a discriminator cavity which acts as the frequency reference. The cavity is a right circular cylinder operating in the TE_{111} mode. The electrical field induced by the input may be considered to be a vector and hence may be resolved into two equal components at right angles to each other in space. These two components are slightly detuned with respect to one another by means of some physical dissymetry. Two outputs respectively couple to each one of the two space orthogonal modes. The response of each mode versus frequency is shown on Fig. 2a. The outputs are detected and fed to a differential amplifier where the two responses are effectively subtracted from one another (Fig. 2b) and where the resultant voltage is amplified. This amplified voltage is then fed back to the klystron reflector and thus closes the feedback stabilizing loop.

Each one of the three components of this stabilizing loop has, associated with it, a particular gain defined as

$$\frac{\text{output}}{\text{input}} .$$

Thus, the discriminator cavity has the gain K_d expressed in volts per mcps; the amplifier has the gain K_a (non-dimensional) and the klystron has the gain K_o, the reflector sensitivity, which is expressed in mcps per volt. The total amplification or loop gain K_t is then equal to the product of the three above amplifications.

If the klystron output frequency tends to change from F_o to F_1 (Fig. 2), a net voltage is produced by the discriminator cavity, and this voltage, after amplification, is then fed back to the klystron in such a way that the potential frequency error $(F_1 - F_o)$ is reduced by the loop gain $K_t + 1$.

Fig. 1 also shows:

A monitoring crystal detector and its associated meter, whose purpose is to indicate whether or not the klystron is oscillating and whether or not its power is peaked;

The klystron tuning which mechanically tunes the oscillator beyond its electronic tuning range;

The reflector voltage control which sets the dc voltage level out of the amplifier;

The frequency control which mechanically tunes the dual mode reference cavity.

II. Detailed Theory of Operation

A. The Feedback Loop

Fig. 3 attempts to picture the feedback circuit in a schematic and conventional way.

The symbols are defined as follows:

Reference cavity frequency	F_o
Klystron open loop frequency	F_1
Open loop frequency error	f_1
Klystron closed loop frequency	F_2
Closed loop error frequency	f_2
Correction frequency	F_3
Cavity gain	K_d mc/volts
Amplifier gain	K_a
Klystron gain	K_o volt/mc
Total gain	K_T
Voltage out of cavity	E_{1c}
Voltage to repeller	E_{2c}
Voltage out of amplifier (open loop)	E_{20}

Note that each physical element fulfills several functions. Thus, the cavity compares an incoming frequency to its own resonant frequency and multiplies the difference by the gain K_d. The amplifier supplies gain and also an undesired voltage source E_{20}. The klystron contains an amplifier of gain K_o; it also adds to its "free funning" frequency F_1, the output frequency of its amplifier, F_3, to produce F_2.

From Fig. 3 and the definitions:

$$F_1 = F_o + f_1 \tag{1}$$

$$f_2 = F_2 - F_o \tag{2}$$

$$E_{1c} = K_d f_2 \tag{3}$$

$$E_{2c} = E_{20} + K_a K_d f_2 \tag{4}$$

$$f_{3c} = K_o E_{20} + K_a K_d K_o f_2$$

$$= K_o E_{20} + K_T f_2 \tag{5}$$

$$F_2 = F_1 - f_{3c}$$

$$= F_1 - (K_o E_{20} + K_T f_2) \tag{6}$$

From (6), (2) and (1):

$$F_2 = F_o + f_1 - [K_o E_{20} - K_T(F_2 - F_o)]$$

$$= (1 + K_T)F_o - K_o E_{20} - K_T F_2 + f_1$$

That is,

$$\boxed{F_2 - F_o = \frac{K_o E_{20}}{1 + K_T} + \frac{f_1}{1 + K_T}} \tag{7}$$

From (4) and (7):

$$E_{2c} = E_{20} + K_a K_d (F_2 - F_o)$$

$$= E_{20} - \frac{K_T E_{20}}{1 + K_T} + \frac{K_a K_d f_1}{1 + K_T}$$

Or

$$\boxed{E_{2c} = \frac{E_{20}}{1 + K_T} + \frac{K_T}{1 + K_T} \frac{f_1}{K_o}} \tag{8}$$

Finally, from (2) and (7):

$$\boxed{f_2 = \frac{-K_o E_{20}}{1 + K_T} + \frac{f_1}{1 + K_T}} \tag{9}$$

Eqs. (8) and (9) suggest the following remarks:

a) f_2, the closed loop error, is made up of two terms. One of them is to be attributed to the amplifier open-loop modulation or dc voltage, E_{20} (E_{20}, E_{2c}, E_{1c}, f_1, etc., may be dc or ac). The other term may come in from the fact that the klystron is not mechanically set to the frequency F_o, (dc condition), or from filament modulation, microphonics, thermal currents, etc., (ac conditions). In both cases, the open loop error is reduced by the factor $(1 + K_T)$.

In practice — and as long as the loop is operating — mechanically tuning the klystron frequency results in an effective frequency change only $(1 + K_T)$ smaller than under open loop conditions. In the same way, attempting to change the amplifier output level results in an effective change only $(1 + K_T)$ smaller than under open-loop conditions.

b) E_{2c}, the input to the klystron reflector, is likewise made up of two terms. However, under actual conditions,

$$\frac{E_{20}}{1 + K_T} \ll \frac{K_T}{1 + K_T} \frac{f_1}{K_o}$$

so that

$$E_{2c} \approx \frac{K_T}{1 + K_T} \frac{f_1}{K_o} \approx \frac{f_1}{K_o} \qquad (8')$$

and from (9) and (8):

$$f_2 \approx \frac{-K_o E_{20}}{1 + K_T} + \frac{K_o E_{2c}}{1 + K_T}$$

or

$$f_2 \approx \frac{K_o (E_{2c} - E_{20})}{1 + K_T} \approx \frac{K_o}{K_T} (E_{2c} - E_{20}) \qquad (9')$$

Thus, under closed loop conditions, the ripple voltage measured at the reflector is essentially

$$\frac{f_1}{K_o}.$$

When the loop is open, the voltage measured is E_{20}. From these two measurements, the greatest error f_2, which occurs when E_{2c} and E_{20} add up in phase is:

$$f_2 \max = \frac{K_o}{K_T} \left(|E_{2c}| + |E_{20}| \right) \qquad (10)$$

where E_{20} is the open loop ripple
and E_{2c} is the closed loop ripple.

B. Discriminator Cavity and rf Considerations

The dual mode discriminator cavity is a right cylinder operating in the TE_{111} mode. The size of the cavity, and hence its resonant frequency, is varied by the axial motion of a plunger. In the X-Band range where waveguide is used, the cavity inputs and outputs are iris-coupled through the bottom of the cavity and the smaller side of the waveguide (Fig. 4). At resonance, the electric field induced by the input may be considered to be a vector and hence may be resolved into two equal components at right angles to each other. Fig. 4 shows how each component of the vector is coupled to a different output.

At S-Band, input and outputs are coupled by means of small loops. Here again, the same feed principle applies: the input loop is oriented 45° away from each one of the two output loops, and those in turn are at right angles to each other.

The two component vectors are slightly detuned with respect to each other by means of some discontinuity in the cavity. Several methods may be used. Tuning screws are inserted in the wall of the cylinder and oriented along the respective components of the electric field; a paddle inserted in the plunger serves the same purpose as the screws by being oriented in such a way as to affect one mode more than the other; a longitudinal groove may be machined out of the plunger and oriented in such a way as to affect one mode more than the other. This last method seems to give the best results: there is no moving part, no intermittent contact and no sharp edges, so that the Q's of the two modes are theoretically equal over the frequency range. Also the "detuning," that is, the frequency separation between the two mode resonances, tends to remain more constant than with tuning screws over the whole tuning range.

The impedance presented by the whole cavity at the plane of a detuned short on the input side is that of two parallel resonant circuits in series, having equal Q's and slightly different resonant frequencies. In Fig. 5, R takes into account cavity losses and output loading.

The impedance of the first resonant circuit is

$$Z_1 = \frac{1}{\frac{1}{R} + j\sqrt{\frac{C}{L}}\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)}$$

Let its normalized value be

$$z_1 = \frac{1}{1 + jx} \tag{11}$$

where R is equal to the characteristic impedance of the line and x is a function of frequency. The half power points correspond to $x = \pm 1$ and the bandwidth to $x = 2$.

Let the impedance of the second resonant circuit be

$$z_2 = \frac{1}{1 + j\,(x - x_o)} \tag{12}$$

where x_o corresponds to the frequency difference between the two resonances. Let

V_{in} = incident reference voltage wave

z_t = normalized total impedance at the input

Γ = reflection coefficient at the plane of detuned short

V = voltage at the plane of the detuned short

P = power dissipated in R

The following conditions are present:

$$V = (1 + \Gamma) V_{in} = [1 + \frac{z_t - 1}{z_t + 1}] V_{in}$$

$$= [\frac{2z_t}{1 + z_t}] V_{in} \tag{13}$$

$$V_1 = V \frac{z_1}{z_t} + \frac{2z_1}{1 + z_t} V_{in} \tag{14}$$

But

$$z_t = z_1 + z_2 = \frac{1}{1 + jx} + \frac{1}{1 + j(x - x_o)}$$

$$= \frac{2 + j(2x - x_o)}{1 - x(x - x_o) + j(2x - x_o)} \tag{15}$$

From Eqs. (14) and (15):

$$V_1 = \frac{2[1 + j(x - x_o)]}{3 - x(x - x_o) + j2(2x - x_o)} V_{in}$$

and

$$|V_1| = \frac{2[1 + (x - x_o)^2]^{1/2}}{\{[3 - x(x - x_o)]^2 + 4(2x - x_o)^2\}^{1/2}} V_{in} \tag{16}$$

By a similar derivation

$$|V_2| = \frac{2[1 + x^2]^{1/2}}{\{[3 - x(x - x_o)]^2 + 4(2x - x_o)^2\}^{1/2}} V_{in} \tag{17}$$

Note that Eqs. (11) and (12) assume one particular type of coupling, that is, a coupling that will make $Z_1 = Z_2 = 1$ at resonance. Note also that with this assumption

$$Z_1 = \frac{2}{1 + \frac{x_o^2}{4}} \quad \text{when} \quad x = \frac{x_o}{2}$$

and $z_1 = 1$ when $x_o = 2$, (see Fig. 5). Thus, it is apparent that the peak separation, x_o affects the loading at resonance and the input bandwidth, and that, in particular, the cavity is matched only when $x_o = 2$ (bandwidth of one mode) with the assumption that $z_1 = 1$ at resonance of the first tank circuit.

For simplicity sake, we shall keep the above conditions and investigate the shapes and relationships in the discriminator curves as x_o is varied.

If the crystals are assumed to operate in the square law region, then the signals out of each crystal are proportional respectively to $|V_1|^2$ and $|V_2|^2$.

The equation of the discriminator curve is given by: $D = |V_1|^2 - |V_2|^2$, the subtraction being effected by the differential amplifier, or

$$D = \frac{4(x_o^2 - 2xx_o)\, V_{in}^2}{[3 - x(x - x_o)]^2 + 4(2x - x_o)^2} \tag{18}$$

The slope at $\frac{x_o}{2}$, the cross-over point is

$$S = \frac{-8x_o V_{in}^2}{(3 + \frac{x_o^2}{4})^2} \tag{19}$$

and this slope is maximum for $x_o = 2$, that is, when the peak separation is equal to the bandwidth of either one of the two resonant modes. For this value of x_o,

$$S\ max = \frac{-V_{in}^2}{1} \quad \text{or} \quad \frac{-2V_{in}^2}{\Delta f} \tag{20}$$

The value of x_o is not extremely critical. Thus for $X_o = 1$, $S = .76\ S_{max}$; for $x_o = 4$, $S = .65\ S_{max}$. This property is very valuable for wide tuning.

From Eq. (20) it appears that the slope is proportional to the input power and inversely proportional to the bandwidth. Ideally, with a given power source, the way to maximize Eq. (10) is to make the unloaded Q of each mode, Q_u, as high as possible and adjust the input and output couplings in such a fashion that Q_L, the loaded Q, is 3 times Q_u so that as much power is dissipated in the cavity as is dissipated in the crystals. Needless to say, it is impossible to match the crystals over a wide band and from a practical point of view it is not even desirable to attempt to do so. Any slight dissymetry in the two outputs becomes very critical as one of the crystal outputs approaches resonance because of frequency pulling on the corresponding mode.

Another practical design consideration is the fact that the discriminator cavity must be decoupled from the klystron oscillator, and the greater Q_L, the greater the amount of decoupling required. The klystron must operate satisfactorily under small changes of reflector voltages and regardless of the load impedance presented by the cavity. Most of all, the klystron must not be "pulled" by the cavity because if two corrections are applied simultaneously by pulling and by electronic means, low frequency oscillations occur.

Finally, the effect of the discriminator cavity in the feedback loop is to introduce two phase lags at a frequency corresponding to half the bandwidth of each individual mode. Increasing Q_L brings these phase shifts to a lower frequency with the ultimate result that the loop gain must be decreased to prevent high frequency oscillations. Design must take all those facts into account and results in a compromise between high Q's and maximum power in the outputs for gain; low Q's for response bandwidth and correct peak separation; reactive crystal holder input impedance for tunability; correct amount of decoupling to prevent excessive pulling but still allow enough power transferred.

C. Differential Amplifier and Feedback Considerations

A block diagram of the differential amplifier is shown on Fig. 6. It consists of two stages of differential amplification using the cascode arrangement for low input noise and high gain. Each stage contains either a high value of cathode resistance or a constant current tube to provide high, and consequently almost equal, "common mode" gain so that changes in parameters are not reflected in the differential output. Amplitude modulation on the inputs has practically no effect on the output. A voltage level changing stage adds a constant voltage to one of the differential outputs and the resultant voltage is applied to a cathode follower which supplies the klystron reflector.

Since the system is closed upon itself, gain and phase shifts must be carefully controlled to prevent the system from oscillating. As mentioned previously, the discriminator cavity introduces two phase shifts at a frequency corresponding to half the bandwidth of each mode. Other unavoidable phase shifts are those caused by the combination of plate resistance and stray plate to ground capacitance for the first stage, plate resistance and stray plate to ground capacitance for the second stage, and $1/g_m$ with stray cathode to ground capacitance for the cathode follower stage. Were no corrective networks introduced, the system would oscillate when the total phase shift is -180^o because the gain is still greater than 1. (This oscillation must not be confused with that resulting from "pulling" of the klystron.) To remedy the situation, the plate load resistance of each differential amplification stage is split into two parts, R_1 and R_2, and C_1 is introduced between the junction point and ground. Typically, R_1 is 100 times greater than R_3. The result of these corrective networks is twofold: first, the gain is reduced by the ratio of $\frac{R_1}{R_2}$

after the corner frequency determined by $R_1 C_1$; secondly, the combination of R_2 and C_1 introduces phase lead at the corner frequency

$$\frac{1}{2\pi R_2 C_1} .$$

As a consequence, the gain is less than 1 when the loop phase shift goes down to -180^{o}.

III. Final Remarks

A typical value for the total loop gain K_T is about 10,000. The amplifier gain is 10,000 up to a frequency of 600 to 800 cps where the gain starts falling off at 12 db per octave. The product of the cavity gain K_d and klystron gain K_o tends to remain constant regardless of the frequency band under consideration. For instance, at S-band, a typical value of K_o is .5 mcps per volt and of K_d is 2 volts per mcps, while at X-band, K_o is 2 mcps per volt and K_d is .5 volts per mcps. Since the close loop error frequency, f_2, is proportional to K_o (Eq. 10), the relative degree of stability is the same at X or S-band.

Although a loop gain of 10,000 with a bandwidth of 700 cps is close to the optimum value with a necessary margin of safety (greater gains are possible at the expense of bandwidth), the stability may be increased by minimizing disturbances on the klystron such as vibrations, thermal fluctuations, and power supply ripple. However, noise in the crystals and in the amplifier imposes a threshold of frequency stability of about one part in 10^9.

Tunability of the system is typically 15 per cent of the midband frequency.

Frequency stability is checked in the following manner: the outputs of two stabilized microwave oscillators are beat together in a crystal mixer. Suitable decoupling is provided between the two oscillators to insure that no pulling or "injection locking" occurs and that no oscillations are set in. One cycle of the beat note is displayed on an oscilloscope. The beginning of the cycles are synchronized, but the ends present a spread (Fig. 7). The frequency stability is computed as follows: the mean beat frequency f_b is carefully determined by means of a calibrating sine wave. The spread after one cycle t_d is likewise carefully determined. The peak to peak frequency deviation of the beat note is $f_b \frac{t_d}{t_b}$. The beat frequency f_b must be as low as possible to make $\frac{t_d}{t_b}$ large enough for accurate measurement. However $f_b \frac{t_d}{t_b}$ is correct only if the carrier frequency f_b is at least ten times higher than either the frequency deviation or the rate of frequency deviation.

The two oscillators are supplied from different power lines (60 cps on one and 400 cps on the other one). They are also physically separated by as great a distance as possible. The frequency disturbances are different and have no common factors. Hence, if a picture is taken of the face of the oscilloscope with an exposure time longer than the longest period of frequency disturbance (1/120 second), the worst possible instability during the exposure time is automatically recorded in the picture.

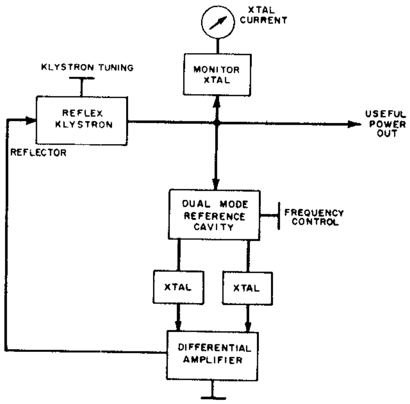

Fig. 1 - Block diagram of stabilized microwave oscillator.

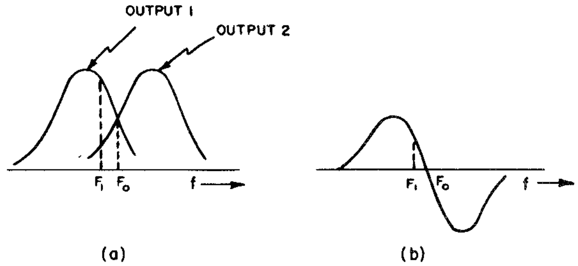

Fig. 2 - Output from each crystal and resultant discriminator curve.

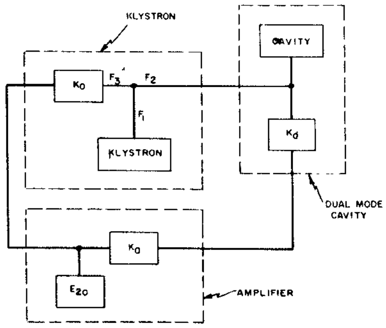

Fig. 3 - Feedback diagram of stabilized microwave oscillator.

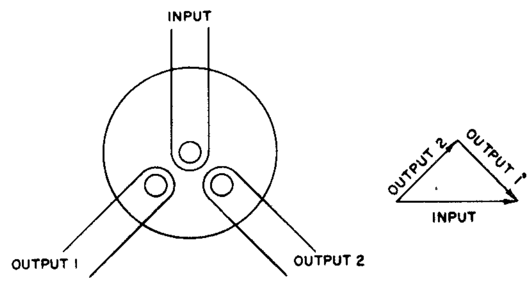

Fig. 4 - Bottom view of dual mode cavity and feed system.

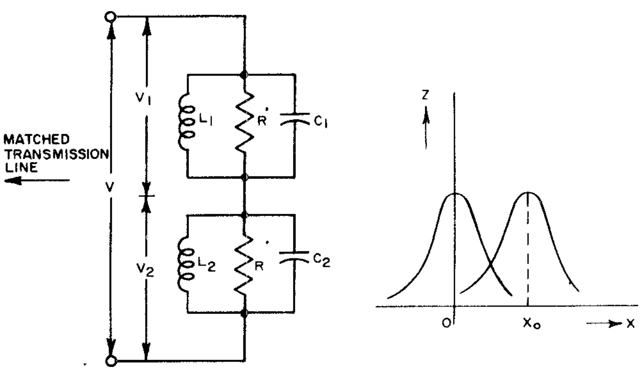

Fig. 5 - Dual mode cavity equivalent circuit.

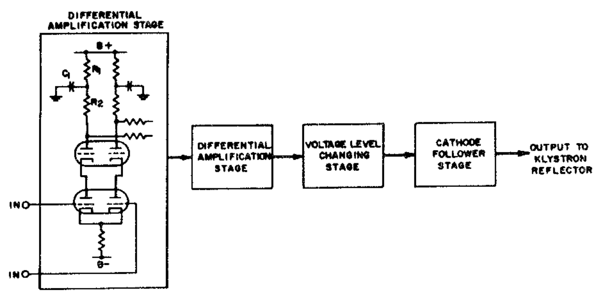

Fig. 6 - Block diagram of differential amplifier.

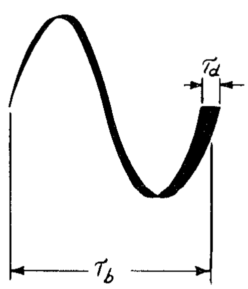

Fig. 7 - Aspect of beat note on oscilloscope.